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4-D GUIDANCE OF STOL AIRCRAFT IN THE TERMINAL AREA

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SUMMARY

The primary objective of advanced STOL aircraft is the improvement of the nation's air transportation system by the elimination of delays and congestions associated with today's air travel. A new guidance technique, referred to as 4-D guidance, is being developed to achieve this objective. The 4-D guidance technique synthesizes complex three-dimensional flight paths from a minimum set of input data and flies the aircraft along the paths according to a prespecified time schedule. The two major elements of a 4-D guidance system are the trajectory synthesizer and the control law. Inputs to the trajectory synthesizer are the three-dimensional coordinates of way points, the turning radii, the speed ranges, the acceleration limits, and the arrival times at time control way points. First, the three-dimensional trajectory is computed by using circular arcs and straight lines. Then the airspeed profile, compensated for wind, is calculated to achieve the desired arrival times. The synthesized trajectory is stored as a time sequence of reference states which the aircraft is forced to track by using a linear feedback law.

INTRODUCTION

If advanced STOL aircraft are to become viable elements of the nation's air transportation network, their unique performance characteristics must be exploited to the fullest possible extent. Most notable among these characteristics are the aircraft's ability to make tight turns, steep climbouts and approaches, and to fly

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over a wide range of airspeeds. Consequently, STOL aircraft will possess much greater flexibility in the terminal area than CTOL aircraft. On the other hand, the performance of STOL aircraft may be limited by two important factors: the aircraft's acceleration/deceleration capability, and the effects of wind. Some existing STOL aircraft are slow to decelerate at some speeds and thus require long deceleration periods in the terminal area. Since these aircraft will be able to fly at very low speeds, on the order of 60 to 65 knots, the effects of wind can become extremely important in predicting arrival times at various key points along the route. In light of these performance characteristics and limitations, the following questions have arisen: Can STOL aircraft follow very demanding ATC instructions? Can they fly complex three-dimensional trajectories in the terminal area? Are they able to maintain an accurate time schedule along their flight path? To answer these questions, a technique is needed to play the characteristics of any potential STOL aircraft against these terminal operating constraints. Such a technique will enable us to determine whether a potential STOL aircraft can operate within these constraints and, if not, to identify STOL aircraft deficiencies to aircraft designers.

It seems likely that advanced short take-off and landing (STOL) aircraft will be equipped with guidance and navigation systems that satisfy far more stringent requirements than those currently used in conventional (CTOL) aircraft. Since these requirements are not yet clearly defined, investigations of the guidance problems for STOL aircraft under the most demanding circumstances were made. Although this approach places a significant burden on the aircraft and onboard guidance system, it ensures that the results obtained will not be invalidated by future developments in ground and airborne system design.

It has been assumed that STOL aircraft will (1) fly curved climbouts and approaches using a microwave landing system; (2) operate in narrow airspace corridors to avoid CTOL traffic, buildings, and other obstacles; and (3) perform noise-abatement maneuvers. The guidance system that meets these requirements is referred to as a 3-D guidance system, and it involves the precise description of complex curved flight paths and the control system to fly the aircraft along the paths.

Precise path control alone, however, is not sufficient to achieve the principal goal of a STOL transportation system, which is the elimination of, or at least the substantial reduction in, undue delays and congestions. This goal can be accomplished by increasing the landing rates at airports; this increase, in turn, implies a greater precision in delivering aircraft to the runway threshold. In the current manual system the accuracy with which aircraft can be delivered to the runway is approximately ±15 sec. It has been suggested that this time can be reduced to about ±2 sec by spacing aircraft accurately with less vectoring and by minimizing pilot-controller communications. Both of these requirements can be fulfilled by achieving precise time control of the aircraft along its flight path. The guidance system that accomplishes this control is referred to as 4-D guidance. It is a guidance technique consisting of two major elements: the 4-D reference trajectory synthesis, and the control law to fly the aircraft along the reference trajectory. It must be emphasized that there is a very important, intrinsic difference between these two elements. By 4-D reference trajectory synthesis is meant the generation of a precise 3-D flight path and a feasible speed profile along the path. Note that this process is a completely open-loop, or predictive, process in the sense that it takes place before the aircraft arrives at the initial point of the trajectory. The action of the control law, on the other hand, is a closed-loop process going on in real time as the aircraft is tracking the reference trajectory. It is the combined open-loop—closed-loop process that is referred to as a 4-D guidance system.

The purpose of this paper is to describe the 4-D guidance system that was developed at NASA Ames Research Center, and which will be flight tested in a STOL aircraft. Some of the crucial design considerations are discussed, and a brief description of the algorithms for trajectory synthesis and control law are given. Finally, some simulation results are presented to indicate the performance of the system.

DESIGN CONSIDERATIONS AND INTERACTION WITH AIR TRAFFIC CONTROL

Chief criteria for selecting the 3-D path are STOL terminal area maneuver requirements and simplicity of computation. These criteria are met by synthesizing the 3-D path from geometrically simple elements. In the horizontal plane these elements consist of segments of circles and straight lines. Complex flight paths are obtained by interconnecting several line segments and sections of circles with different radii. Paths constructed in this manner can yield minimum time trajectories as discussed in references 1 to 3. The vertical trajectory is synthesized from sections of constant flight-path angle. The complete three-dimensional flight path is then obtained by requiring the aircraft to fly the vertical profile along the ground track determined by the previously computed horizontal trajectory. A critical problem in implementing this synthesis procedure is minimizing the pilot work load in entering a trajectory. As explained in the next section, this problem is solved by using way points to specify the trajectory.

After the three-dimensional path has been established, the desired position of the aircraft along the path as a function of time is determined from considerations of air traffic control. Generally, more than one aircraft will be flying along the 3-D flight path or will be merging with the path at certain of its points referred to as merging points. This effect is illustrated in figure 1. Aircraft (AC) on the two approach routes merge in the vicinity of the approach gate. In the current air traffic control (ATC) system the final approach controller is responsible for merging aircraft at this point. He does this by observation of the aircraft on the ATC radar and by issuing speed and vectoring instructions to the pilots. The delays, inaccuracies, and other limitations of this manual controller—pilot control loop yield a broad envelope of aircraft trajectories between the feeder fix and the gate, shown as the hatched area in figure 1. This manual technique can deliver aircraft to the approach gate with a time accuracy of approximately ±15 sec. It is well-known that the time control accuracy influences the required spacing on final approach, which, in turn, determines the landing capacity of the runway (ref. 4). Through the use of 4-D guidance techniques, the accuracy of time control can be greatly increased; thus, a basis for achieving higher landing rates and greater automation in the control of terminal area traffic is provided. Instead of the controller issuing vectoring and speed commands to space aircraft, the air traffic control system specifies only the desired arrival times at a small number of points along the terminal approach route and leaves the burden of computing aircraft control inputs to achieve these times to the onboard system.

From a knowledge of the 3-D path and the desired arrival times at specified points on the path, the 4-D guidance system computes the required airspeed profile. The airspeed computation algorithm must consider the minimum and maximum permissible airspeed, the aircraft's acceleration and deceleration capability, the landing approach speed, and the effect of winds. Also, in specifying arrival times, air traffic control will use only limited knowledge of the aircraft's performance capabilities. Therefore, the algorithm must first determine the feasibility of the specified arrival times by comparing them with the true minimum and maximum times the aircraft can achieve without deviating from the 3-D path. If the specified arrival times cannot be achieved, air traffic control must be requested to reassign arrival times or permit delaying maneuvers such as holding and/or path stretching.

In the preceding discussion of arrival time assignment, the question left unanswered was how air traffic control will generate the arrival times to be assigned to each aircraft. To clarify this question, consider an aircraft equipped with a 4-D guidance system which has just arrived at one of the feeder fixes in figure 1. It is assumed that the aircraft had previously been cleared to proceed toward the feeder fix. Those aircraft currently flying between the feeder fixes and the runway were previously assigned arrival times at the gate and the touchdown point. Given the schedules of these aircraft and, for the new aircraft, an estimate of the minimum and maximum times to the gate and from the gate to the touchdown point, a technique is needed to specify feasible times at the gate and at the touchdown point such that separation standards between aircraft are satisfied and that the aircraft lands in minimum time. A general algorithm to calculate such times has been described in reference 5.

SPECIFICATION OF 3-D PATH

The problem of specifying and calculating the 3-D path is divided into two problems solved in sequence. First the projection of the trajectory in a horizontal plane is computed from an analysis of the way-point coordinates and the desired turning radii. Then the known arc length of the horizontal trajectory together with the altitude difference between adjacent way points is used to determine the flight-path angle and, therefore, the altitude profile between adjacent way points.

A crucial part in the calculation of horizontal trajectory parameters is the interpretation of way points. This part is explained with the help of an example trajectory shown in figure 2. The trajectory begins at lift-off and terminates at touchdown, way point 7 (WP7). Although this trajectory can be flown by a STOL aircraft, its shape has no significance beyond illustrating the construction procedure. All parts of the trajectory consist of segments of straight lines and circles. The basic problem to be solved can be stated as follows: What is the essential input information that the pilot must provide to the airborne computer in order to generate this trajectory uniquely? The solution lies in the definition of two types of way points, which are referred to as the ordinary and final heading way points.

Ordinary Way Points

An ordinary way point is exemplified in figure 2 by way points, 2, 4, and 5 (WP2, WP4, and WP5). Its location is defined by the intersection of two straight lines. The two lines are connected by an arc of a circle tangent to both lines. Thus, the sharp corner at the way point is rounded to obtain a trajectory the aircraft can fly. The radius of the circle used in rounding the corner can either be explicitly specified by the pilot as in this example or it can be implicitly determined from a maximum bank angle constraint ϕ_{max}. For a given ϕ_{max}, the minimum turning radius R_{min} depends on the maximum ground speed $V_{g,max}$ that can be attained in a 360° turn and is

$$R_{min} = \frac{V_{g,max}^2}{g \tan \phi_{max}} \quad (1)$$

where g is the acceleration of gravity. The maximum ground speed is the sum of the maximum airspeed and the magnitude of the wind vector.

Throughout this paper, the runway-centered coordinate system is used to specify points in the plane. The origin of this system is at the touchdown point, way point 7, the X-axis points in the landing direction and the Y-axis points to the right when facing in the landing direction. Heading angles are measured clockwise from the direction of the positive X-axis.

Suppose the pilot has entered the x- and y-coordinates of a sequence of ordinary way points together with the turning radii to be used in rounding the corners at way points. From this information the onboard computer calculates various parameters defining the trajectory. It is convenient to compute these parameters successively, starting with the last way point to be flown through and ending with the first one. For this purpose figure 3 shows the trajectory between way points i - 1 and i + 1 and also defines various quantities used in the calculation. It is assumed that the calculations from way point i + 1 to the last way point have been completed. These calculations yielded the quantities l_{i+1}, x_{i+1}, x_{Pi+1}, and ψ_{i+1} which together with x_i, y_i, R_i, x_{i-1}, and y_{i-1} are used to obtain the parameters for the ith way point. The heading of the straight-line segment between way points i - 1 and i is ψ_i, given by the relation

$$\psi_i = \text{arc tan} \frac{y_i - y_{i-1}}{x_i - x_{i-1}} \qquad \left(-180° < \psi_i \leqq 180°\right) \qquad (2)$$

The heading change ψ_{ti} in the circular segment near way point i is

$$\psi_{ti} = \underset{180°}{\text{mod}} \left(\psi_{i+1} - \psi_i\right) \qquad \left(-180° < \psi_{ti} \leqq 180°\right) \qquad (3)$$

where the direction of the turn is to the right for $\psi_{ti} > 0$ and to the left for $\psi_{ti} < 0$. Next, calculate the quantity b_i shown in figure 3.

$$b_i = R_i \tan \left|\frac{\psi_{ti}}{2}\right| \tag{4}$$

The length of the straight segment d_{i+1} is then

$$d_{i+1} = l_{i+1} - b_i \qquad \left(l_{i+1} \geqq 0\right) \tag{5}$$

If d_{i+1} from equation (5) is less than zero, adjacent turns overlap, the computation of the trajectory must stop, and the pilot is given a diagnostic message, such as "way points i and $i + 1$ are too close." The coordinates for the end of the turn and the center of the turn are

$$x_{Qi} = x_i + b_i \cos \psi_{i+1} \tag{6}$$

$$y_{Qi} = y_i + b_i \sin \psi_{i+1} \tag{7}$$

$$x_{Ri} = x_{Qi} \mp R_i \sin \psi_{i+1} \tag{8}$$

$$y_{Ri} = y_{Qi} \pm R_i \cos \psi_{i+1} \tag{9}$$

where the upper sign is chosen for a right turn and the lower sign for a left turn. Next, the distance l_i is calculated:

$$l_i = \sqrt{\left(x_{i-1} - x_i\right)^2 + \left(y_{i-1} - y_i\right)^2} - b_i \qquad \left(l_i \geqq 0\right) \tag{10}$$

If l_i is less than zero, the calculation cannot continue and the pilot is given a diagnostic message such as "way points i - 1 and i are too close." If $l_i \geqq 0$, the iteration is completed by calculating the coordinates of the beginning of the turn:

$$x_{Pi} = x_i - b_i \cos \psi_i \tag{11}$$

$$y_{Pi} = y_i - b_i \sin \psi_i \tag{12}$$

These iterations are continued until the first way point is reached. Since the way points entered do not always yield a flyable trajectory, onboard calculation of the trajectory generally will require a system that permits the pilot to correct errors after the system has issued a diagnostic message.

Final Heading Way Points

Final heading way points are illustrated in figure 2 by way points 3, 6, and 7. Instead of rounding the corner at the intersection of two lines, the trajectory for this type passes through the way point at the instant the turn toward the next way point has been completed. Thus, the aircraft begins its flight along the straight-line segment exactly over the way point. There are two reasons for introducing this type of way point. First, it simplifies the specification of some trajectory segments, such as the turn at way point 3, which contains more than 180°. Recall that the turn at an ordinary way point is limited to less than 180°. Second, this type is required if the arrival time at the way point is specified. Specification of arrival time at an ordinary way point lacks precision since the way-point coordinates themselves do not fall on the trajectory. By requiring all aircraft that are merging to fly through a point on the merging path with a common heading, the assignment of arrival times at the merging way point can be used to achieve precise spacing of aircraft.

In the specification of a trajectory, ordinary and final heading way points can be alternated in arbitrary fashion. The general procedure for calculating the trajectory parameters is illustrated in figure 4 with a final heading way point i embedded between way points $i - 1$ and $i + 1$ of arbitrary type. As before, the trajectory is computed backwards from the last way point. The final heading ψ_{i+1} to be achieved at way point i was previously determined in the calculation for way point $i + 1$ and is therefore a known quantity. It is evident from figure 4 that the desired final heading at way point i can be achieved with two trajectories, one ending with a left turn, the other with a right turn. The criterion for selection is to choose the one with the shorter path length between way points $i - 1$ and i. To make the selection, the coordinates of the centers of the two turns are calculated:

$$x_{Ri} = x_i - R_i \sin \psi_{i+1} \tag{13}$$

$$y_{Ri} = y_i + R_i \cos \psi_{i+1} \qquad \text{(Right turn)} \tag{14}$$

$$x_{Li} = x_i + R_i \sin \psi_{i+1} \tag{15}$$

$$y_{Li} = y_i - R_i \cos \psi_{i+1} \qquad \text{(Left turn)} \tag{16}$$

Then, the distances squared from way point $i - 1$ to each center are

$$d_{Ri}^2 = \left(x_{Ri} - x_{i-1}\right)^2 + \left(y_{Ri} - y_{i-1}\right)^2 \tag{17}$$

$$d_{Li}^2 = \left(x_{Li} - x_{i-1}\right)^2 + \left(y_{Li} - y_{i-1}\right)^2 \tag{18}$$

From the geometry of the construction in figure 4, it can be seen that the trajectory with the shorter path length also has associated with it the shorter of the two distances d_{Ri} and d_{Li}. Thus, the right-turn trajectory is chosen if d_{Ri} is greater than d_{Li}. If the way point $i - 1$ lies on the line determined by way points i and $i + 1$, both trajectories have the same length and the direction of the turn, if a turn is required, must be selected on the basis of another criterion. The next step is to determine whether way point $i - 1$ lies inside or outside the circle used to define the turn. Suppose the right turn was previously selected; then the way point lies inside if $d_{Ri}^2 < R_i^2$ and outside or on the circle if $d_{Ri}^2 \geq R_i^2$. If this former case is true, the trajectory is not feasible, further computation of the trajectory stops, and the pilot is given the message "way points i and $i - 1$ are too close." In the latter case, the calculation continues with the computation of the heading ψ_{dRi} of the directed line d_{Ri} if a right turn is required or the heading ψ_{dLi} of the directed line d_{Li} if a left turn is required:

$$\psi_{dRi} = \text{arc tan} \frac{y_{Ri} - y_{i-1}}{x_{Ri} - x_{i-1}} \qquad \text{(Right turn)} \tag{19}$$

$$\psi_{dLi} = \text{arc tan} \frac{y_{Li} - y_{i-1}}{x_{Li} - x_{i-1}} \qquad \text{(Left turn)} \tag{20}$$

Next, the angle ψ_{Ri} is computed for a right turn:

$$\psi_{Ri} = \arc \sin \frac{R_i}{d_{Ri}} \tag{21}$$

or ψ_{Li} for a left turn:

$$\psi_{Li} = \arc \sin \frac{R_i}{d_{Li}} \tag{22}$$

The heading ψ_i is then

$$\psi_i = \frac{\text{mod}}{180^\circ} \psi_{dRi} - \psi_{Ri} \tag{23}$$

if the trajectory contains a right turn and

$$\psi_i = \frac{\text{mod}}{180^\circ} \psi_{dLi} - \psi_{Li} \tag{24}$$

if it contains a left turn. The length d_i of the straight-line segment from way point $i - 1$ to the beginning of the turn is

$$d_i = \sqrt{d_{Ri}^2 - R_i^2} \qquad \text{(Right turn)} \tag{25}$$

$$d_i = \sqrt{d_{Li}^2 - R_i^2} \qquad \text{(Left turn)} \tag{26}$$

Finally, the heading change ψ_{ti} in the turn is obtained from the difference between ψ_i and ψ_{i-1}, and the coordinates of the beginning of the turn to way point i are

$$x_{Pi} = x_{i-1} + d_i \cos \psi_i \tag{27}$$

$$y_{Pi} = y_{i-1} + d_i \sin \psi_i \tag{28}$$

Technique of Flying to the First Way Point

In the preceding discussion, the trajectory from the first to the last way point was synthesized. To complete the synthesis, the trajectory from the aircraft's initial position and heading to the first way point must be constructed.

The construction procedure depends on the type of the first way point. If it is an ordinary way point, the trajectory is constructed on the basis of the rule that the aircraft will turn from its current heading toward the first way point in the direction that minimizes the total path length to the first way point. This criterion is the same as that used in constructing the trajectory from way point i + 1 to way point i, where way point i is of the fixed final heading type. Thus, the procedures of the preceding section apply if the initial heading ψ_o is identified with ψ_{i+1}, the initial position is identified with the coordinates of way point i, and the coordinates of the first way point are identified with those of way point i - 1, and the airplane traverses figure 4 in the opposite sense.

If, however, the first way point is the fixed final heading type, a procedure different from those discussed so far must be used. This guidance problem can be stated as follows: determine a trajectory that starts from a given initial position and heading and leads to a position with a specified final heading. Since this problem has been dealt with extensively in references 1, 2, 3, 6, and 7, only a brief discussion of results is given here. References 1 and 2 give a minimum time solution to this problem, reference 2 deriving the optimum control law. Simplified solutions are discussed in references 3 and 6. In the simplified treatment, the required trajectory consists of a turn, a straight flight, and then another turn. The parameters of the three segments are chosen to satisfy the initial and final conditions of the problem.

Altitude Profile

The calculation of the altitude profile is simple and requires only the determination of a flight-path angle γ_i between way points $i - 1$ and i. Since the horizontal path length between way points is known, the flight-path angle γ_i is given by

$$\gamma_i = \arc\tan \frac{h_i - h_{i-1}}{s_i} \tag{29}$$

where s_i is the horizontal path length between way points i and $i - 1$, and h_i and h_{i-1} are the altitudes specified at way points i and $i - 1$, respectively. In order to compute s_i from the segments of turns and straight lines it is necessary to define at what point on the horizontal trajectory the specified way-point altitude is to be achieved. The rule used is that the way-point altitude must be achieved exactly at the end of the turn for a given way point. This rule is used for both ordinary and final heading way points. The last step in the altitude profile computation is to check whether each γ_i lies within the range of permissible flight-path angles for the aircraft.

It is possible and perhaps desirable to define more complicated altitude profiles between way points. Profiles that minimize a performance function such as the fuel consumed could be valuable. However, the limited size of the airborne computer to be used in flight tests of this guidance system, together with the requirement for onboard computation of the trajectory, does not permit consideration of more complex techniques at this time.

SPEED PROFILE COMPUTATION

Precise time control of aircraft is achieved by determining a feasible speed profile along the already computed 3-D trajectory. A speed profile is said to be feasible if it satisfies the following conditions:

1. The airspeed remains between the minimum and maximum airspeed restrictions imposed along the trajectory.

2. The rate of change of airspeed does not exceed the acceleration/deceleration capabilities of the aircraft.

3. The resulting ground speed yields the desired arrival times at those way points where they have been prescribed by ATC (such way points are referred to as time-controlled way points).

The first two conditions imply that the flying time between any two points on the trajectory is bounded above and below by the minimum and maximum times corresponding to the maximum and minimum airspeeds, respectively. Consequently, if arrival times at the time-controlled way points are assigned arbitrarily, then a feasible speed profile may not exist. In order to insure the existence of a feasible speed profile, ATC assigns arrival times based on the minimum and maximum possible flying times between successive pairs of time-controlled way points.

The determination of a feasible speed profile in the absence of wind would be a relatively simple task, for in this case airspeed and ground speed are identical. In the presence of wind, however, the relationship between airspeed and ground speed is highly nonlinear, and the problem becomes considerably more complex. By assuming a steady wind, the expression for the magnitude of the ground speed $V_g(t)$ at any time t is given by

$$V_g(t) = \sqrt{V_a^2(t) - V_w^2 \sin^2 \psi_w(t)} + V_w \cos \psi_w(t) \tag{30}$$

where $V_a(t)$ and V_w are the magnitudes of the airspeed and windspeed, respectively, and $\psi_w(t)$ is the angle fromthe wind direction to the ground heading, measured positive clockwise. The differential equation governing $\psi_w(t)$ is:

$$\left|\frac{d\psi_w(t)}{dt}\right| = \begin{cases} 0 & \text{(Straight flight)} \quad (31a) \\ \frac{1}{R} V_g(t) & \text{(Circular flight)} \quad (31b) \end{cases}$$

where R is the radius of turn. Exact analytic expressions for $\psi_w(t)$ and $V_g(t)$ can be found only in the case of straight flight with constant airspeed. A considerable simplification can be made by assuming that for all t

$$\left(\frac{V_w}{V_a(t)}\right)^2 << 1 \tag{32}$$

Supporting evidence indicates that inequality (32) is a good approximation not only for CTOL but STOL aircraft operations as well. Under this assumption equation (30) can be written as

$$V_g(t) = V_a(t) + V_w \cos \psi_w(t) \tag{33}$$

Before considering the speed profiles for straight and curved flight in more detail, it is necessary to establish certain desirable characteristics for airspeed profiles. In the design of the 4-D guidance system described in this paper, the point of view was adopted that the airspeed profile should be a piecewise linear function of time, that is,

$$V_a(t) = V_a(t_k) + a_k(t - t_k) \quad (t_k \leqq t \leqq t_{k+1},\ k = 0,1\ldots) \tag{34}$$

where a_k is the constant value of acceleration/deceleration in the interval (t_k, t_{k+1}). Furthermore, changes in airspeed should occur only at a few places along the trajectory, preferably at those points where the minimum and maximum admissible airspeeds change. These requirements were dictated by considerations of passenger comfort, pilot workload, and simplicity of implementation. A typical airspeed profile possessing these characteristics is shown in figure 5.

Straight Flight

Let the desired airspeed along a straight flight segment of length d_k be given by equation (34). Then the analytic expressions for ψ_w, V_g, and t_{k+1} are

$$\psi_w(t) = \psi_w(t_k) \tag{35}$$

$$V_g(t) = V_a(t_k) + a_k(t - t_k) + V_w \cos \psi_w(t) \tag{36}$$

$$t_{k+1} = t_k + \frac{-V_g(t_k) + \sqrt{V_g^2(t_k) + 2a_k d_k}}{a_k} \tag{37}$$

Curved Flight

Let a curved flight segment consist of a circular turn of ψ_{tk} radians with turning radius R_k ($\psi_{tk} > 0$ for a right turn, $\psi_{tk} < 0$ for a left turn). If the desired airspeed along the circular arc is given by equation (34), then no analytic expressions can be found for ψ_w, V_g, and t_{k+1}. Since numerical integration for determining the speed profile would be prohibitive, a different approach is needed. It so happens that equation (31b) becomes integrable if the airspeed has the following form:

$$V_a(t) = V_a(t_k) + a_k\left(1 + \frac{V_w \cos \psi_w(t)}{V_a(t_k)}\right)(t - t_k) \quad \left(t_k \leqq t \leqq t_{k+1}\right) \tag{38}$$

In this case the analytic expressions for ψ_w, V_g, and t_{k+1} are

$$\psi_w(t) = 2 \tan^{-1}\left\{\frac{C_1}{C_2} \tan\left[\tan^{-1}\left(\frac{C_2}{C_1} \tan \frac{\psi_w(t_k)}{2}\right) + \frac{C_2}{2R_k \operatorname{sgn} \psi_{tk}}\left(t + \frac{a_k}{2V_a(t_k)} t^2\right)\right]\right\} \tag{39}$$

$$V_g(t) = V_a(t_k) + a_k\left(1 + \frac{V_w \cos \psi_w(t)}{V_a(t_k)}\right)(t - t_k) + V_w \cos \psi_w(t) \tag{40}$$

$$t_{k+1} = \frac{V_a(t_k)}{a_k}\left(-1 + \sqrt{1 + \frac{2a_k}{V_a(t_k)} \bar{t}_{k+1}}\right) \tag{41}$$

where C_1, C_2, and $\bar{t}_k$ are defined by

$$C_1 = V_a(t_k) + V_w$$

$$C_2 = \sqrt{V_a^2(t_k) - V_w^2}$$

$$\bar{t}_{k+1} = \frac{2R_k \operatorname{sgn}(\psi_{tk})}{C_2} \left\{ \tan^{-1}\left[\frac{C_2}{C_1} \tan\left(\frac{\psi_w(t_k) + \psi_{tk}}{2}\right)\right] - \tan^{-1}\left[\frac{C_2}{C_1} \tan\left(\frac{\psi_w(t_k)}{2}\right)\right]\right\}$$

(Note that $t_{k+1} = \bar{t}_{k+1}$ if $a_k = 0$.) Although V_a given by equation (38) is not a linear function of time, for values of $V_a(t_k)$ and V_w satisfying inequality (32), $V_a(t)$ turns out to be very nearly linear. This condition is illustrated in figure 6, which shows the airspeed and ground speed along a section of the example trajectory of figure 2.

The only remaining quantity still to be determined is the desired airspeed profile. Since the earliest and latest possible arrival times are achieved by flying the aircraft on the boundaries of the admissible speed ranges, the actual airspeed profile corresponding to an intermediate arrival time must lie between the speed boundaries. (See fig. 5.) The nonlinear relationship between arrival time and airspeed necessitates the use of an iterative procedure for the determination of the desired airspeed profile. Basically, the procedure adjusts the cruising speed level between each pair of time-controlled way points so that the prescribed arrival times are achieved. This adjustment is made by using expressions of the form of equations (37) and (41).

A final remark concerning the wind is in order. In this paper only the case of a steady wind is considered. It is well-known, however, that both the magnitude and direction of the wind are functions of the altitude. If these functions were known, they could be easily incorporated in the speed profile computation.

AN EXAMPLE OF 4-D TRAJECTORY SYNTHESIS

The preceding sections described the techniques used to calculate the two major elements of a 4-D trajectory, the 3-D path and the airspeed profile along the path. These elements must now be assembled to produce the complete reference trajectory consisting of the reference states (position, altitude, and heading) and the reference controls (turning radius, airspeed, and flight-path angle) as a function of time from initial time to final time.

The reference states and commands are calculated by a procedure which makes use of the chosen parameterization of the 4-D trajectory to minimize computer storage. The calculation is done in two steps. In the first step, the 3-D path and the airspeed acceleration time history along the path are used to construct a command table consisting of a sequence of control inputs arranged in chronological order. Since the reference controls are piecewise constant in time, the command table gives the values of the reference controls only at time instants where they change to new constant values. In the second step, the reference states between command times are computed analytically from the initial condition at the command time and the value of the controls during the command interval. Compared with the technique of storing the reference trajectory at a large number of time instants, this technique uses significantly less storage, an important consideration in implementing the technique on an airborne computer.

The example trajectory shown in figure 2 is used to illustrate the technique of 4-D trajectory synthesis described in this paper. The pilot specified the trajectory to the system by entering the data given in table I. In the onboard system the way-point types are replaced by numerical codes. Both the initial position (lift-off) and the touchdown point are treated as final heading way points in synthesizing the trajectory. The initial heading and the runway heading associated with these way points are both 0° in this case. Final heading way point 6 and the touchdown point are time control points with arrival times of 300 and 350 sec, respectively. The wind is assumed to be from 0° at 25 ft/sec. The airspeed range specified for each way point in table I is valid from the end of the turn performed at that way point to the end of the turn performed at the next way point. This procedure requires choosing starting times of decelerations/accelerations such that the airspeed will fall in the next speed range at precisely the end of the turn. These fixed boundary conditions are met by synthesizing the airspeed profile backward from the last way point.

The 4-D command sequence generated for this input is given in table II. The minimum and maximum arrival times at the two time control way points (WP6 and WP7) are given in the headnote of the table. There are 17 command times in this example. Columns 3 to 8 give the states at the command times and columns 9 to 11 the piecewise constant controls between command times. Space limitations prevent giving the equations for computing the reference states between command times. For the same reason the equation for computing the instantaneous bank angle from the airspeed, heading, wind vector, and the turning radius is not given. Bank angle and flight-path angle commands are applied to the aircraft stability augmentation system (SAS) slightly in advance of those given in the table to minimize errors due to the finite rates of these quantities. Computation time of this trajectory on the IBM 360 is 0.5 sec.

PERTURBATION EQUATIONS AND CONTROL LAW

A synthesis procedure having been developed for the 4-D trajectory, which will now be referred to as the reference trajectory, the next step is to design a control law for flying it. The design of a control law for this problem, which is based on the technique described in reference 7, is accomplished by means of a perturbation method. Design of the control law for the altitude channel will not be considered here since this channel is simple with minimal coupling to the other channels.

The nonlinear dynamical equations, from which the perturbation equations are derived, are as follows:

$$\dot{X} = V_a \cos \Psi \tag{42}$$

$$\dot{Y} = V_a \sin \Psi \tag{43}$$

$$\dot{\Psi} = \frac{g(\tan \Phi)}{V_a} \tag{44}$$

where V_a is the airspeed, $\dot{X}$ and $\dot{Y}$ are components of V_a, Ψ is the heading, $\dot{\Psi}$ the heading rate, Φ the bank angle, and g the acceleration of gravity. The wind is assumed to be zero and the flight-path angle γ small so that $\cos \gamma \approx 1$.

The perturbation equations are obtained from the nonlinear equations by expansion in a Taylor series about a moving target reference system as illustrated in figure 7. The origin and positive X-axis of this system at any given time are the reference position and the direction of the aircraft flying the reference trajectory, respectively. In figure 7, X_r, Y_r, and Ψ_r refer to the reference position and heading and X_a, Y_a, and Ψ_a, to the aircraft position and heading in the runway-centered coordinate system. The linear differential equations obtained from a Taylor series expansion in the perturbed quantities X, Y, and Ψ shown in figure 7 are

$$\left.\begin{aligned} \dot{x} &= v_a + \dot{\Psi}_r y \\ \dot{y} &= V_r \psi - \dot{\Psi}_r x \end{aligned}\right\} \tag{45}$$

$$\dot{\psi} = \frac{g}{V_r}\left(\sec^2 \Phi_r\right)\phi - \frac{\dot{\Psi}_r}{V_r} v_a \tag{46}$$

where $v_a = V_a - V_r$ and $\phi = \Phi_a - \Phi_r$. The advantage of deriving the perturbation equations in this target referenced system is that terms involving $\sin \Psi_r$ and $\cos \Psi_r$, which would otherwise appear, are eliminated; thus the perturbation equations for curved trajectories are simplified. Equations (45) and (46) also show that x and y are coupled when $\dot{\Psi}_r \neq 0$, whereas the gain of the ψ channel is inversely proportional to V_r; thus, $\dot{\Psi}_r$ and V_r are parameters which depend on the reference trajectory. A control law for nulling the perturbed quantities will therefore have to contain $\dot{\Psi}_r$ and V_r as parameters.

The control variables of the aircraft for tracking the reference aircraft are the bank angle Φ_a and the speed V_a. A linear model of the combined autopilot and aircraft dynamical response for a bank angle and velocity command system can be approximated by the following equations:

$$\ddot{\phi} = \frac{1}{\tau_\phi}\left(-\dot{\phi} + k_\phi \phi - k_\phi \phi_c\right) \tag{47}$$

$$\ddot{v}_a = \frac{1}{\tau_v}\left(-\dot{v}_a + k_v v_a - k_v v_c\right) \tag{48}$$

where ϕ_c and v_c are the command inputs and ϕ and v_a are the response. The parameters in these two equations, k_ϕ, k_v, τ_ϕ, and τ_v, were deduced by matching the step responses of these equations to those of a currently in-service four-engine jet aircraft with an autopilot and autothrottle. Their numerical values are 0.375 sec^{-1}, 0.167 sec^{-1}, 1.04 sec, and 4.17 sec, respectively. The following control law is chosen for nulling the perturbed quantities x, y, and ψ:

$$\phi_c = - k_{\phi x}\dot{\Psi}_r x - k_{\phi y} y - k_{\phi\psi} V_r \psi \tag{49}$$

$$v_c = - k_{vx} x - k_{vy}\dot{\Psi}_r y \tag{50}$$

Note that equations (49) and (50) contain $\dot{\Psi}_r$ and V_r as parameters. This parameterization has been found to be effective in achieving acceptable performance of the control law for the class of reference trajectories of interest here.

The governing factors for determining the numerical values of the five gains k_{vx}, k_{vy}, $k_{\phi x}$, $k_{\phi y}$, and $k_{\phi\psi}$ are (a) the accuracy of the navigation data, (b) the allowable bank angle and throttle activity for passenger comfort, and (c) the accuracy of following the synthesized reference trajectory. A root-locus analysis of the closed loop system indicates that a good compromise between conflicting requirements (b) and (c) is to use 0.0002 rad/ft for $k_{\phi y}$, 0.004 rad/rad (ft/sec) for $k_{\phi\psi}$, 0.0001 rad/ft (sec/rad) for $k_{\phi x}$, 0.04 (ft/sec)/ft for k_{vx}, and 0.15 (ft/sec)/ft (sec/rad) for k_{vy}. This combination of gain constants was obtained by trial and error by using a root-locus analysis of equations (45) to (50). The roots corresponding to this set of gains yield reasonable frequency and damping for all values of $\dot{\Psi}_r$ from zero to 6°/sec. A root-locus plot of equations (45) to (50) as a function of $\dot{\Psi}_r$ for the choice of gains given here can be found in reference 7.

SIMULATION RESULTS

Figure 8 shows the block diagram of the simulation used to evaluate the guidance system. The general flow of computations in the trajectory synthesis algorithm is indicated inside the block drawn with dashed lines. The final product of the synthesis computations is the command schedule. The reference states and controls generator uses the command schedule, clock time, and the measured wind vector to compute the reference states and controls for each control time interval. An interval of 0.1 second was used in the simulation. By use of the measured wind vector, ground speed and ground heading are converted to reference airspeed V_r and airspeed heading Ψ_r.

The control loop used to fly the aircraft along the reference trajectory is shown in detail. The first step is to compute the perturbed quantities x, y, z, and ψ, which are obtained in the transformations given in the bottom block. These quantities are multiplied by the appropriate gains and are added as required to form the perturbation controls v_c, ϕ_c, and z_c. They are then subtracted from the reference controls to form the autopilot and autothrottle inputs consisting of the command bank angle Φ_c, the command airspeed V_c, and the command altitude rate $\dot{Z}_c$.

A simplified model of the autopilot, autothrottle, and aircraft dynamics was developed especially for use in 4-D guidance and air traffic control simulation studies. A detailed flow chart for this model is given in reference 7. The model consists of a tenth-order dynamic system with hard limits on roll,

roll rate, airspeed, airspeed acceleration, flight-path angle, and flight-path-angle rate. The actual wind vector is also an input to the model. Output quantities are the actual aircraft states. From these quantities the navigation system simulation obtains the measured aircraft states and the estimated wind vector.

A complete analysis of simulation results cannot be given within the length of this paper. Only the response of the control law to track a STOL type reference trajectory in the horizontal plane consisting of a 360° circular segment with radius of 1220 ft and an airspeed of 135 ft/sec will be shown. The reference trajectory, which has a duration of 56.7 sec, is generated with two final heading way points located on the circle as shown in figure 9. This reference trajectory is a severe test of the control law since to fly it requires a reference bank angle of 25°, almost equal to the bank angle limit of 30° used in the aircraft simulation; this angle leaves little bank margin for nulling out errors. Figure 9 also shows the trajectories of the simulated aircraft for two initial conditions and an error in the wind estimate. The position of the reference aircraft and of the simulated aircraft is marked every 10 sec along the trajectories.

Starting from the two initial conditions, the simulated aircraft locks onto the reference trajectory after 30 sec of flight even though a period of bank angle limiting occurs (not shown in fig. 9) while the control law nulls the errors. To evaluate the effect of wind-estimate errors on tracking accuracy, an 8.45 ft/sec (5 knots) constant wind error was introduced. Normally, a wind estimator, which is part of the navigation system, would observe this error to a degree and refine its estimate, but in this case the estimator was disabled. The resulting tracking error is 180 ft at the end of the trajectory and indicates the importance of accurate wind estimates in precision aircraft control.

CONCLUSION

The chief advantage of the approach to 4-D guidance described here is the ability to specify and compute complex trajectories in flight. This feature is a highly desirable one from the pilot's viewpoint. Another advantage is that the technique is not strongly dependent on the aircraft type, since the only aircraft parameters used in synthesizing the trajectories are performance limitations, which are treated as parameters. Furthermore, the guidance technique can be integrated with a ground-based scheduling technique to form a complete air traffic control system. The precision of trajectory control and arrival time achieved with the system provides a solid base for reducing separation requirements and increasing landing rates in future air traffic control system.

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TABLE I. — INPUT QUANTITIES REQUIRED FOR EXAMPLE TRAJECTORY

[Initial heading, 0°; runway heading, 0°; airspeed acc/dec, 1.5 ft/sec^2; wind speed/direction, 25 ft/sec/0°; initial airspeed, 135 ft/sec]

Way-point number	Way-point type	Way-point coordinates, ft			Turn radius, ft	Airspeed range, ft/sec	Time, sec
		x	y	h			
1	Initial position	1,000	0	0	4000	135 to 135	0
2	Ordinary	5,000	0	600	4000	203 to 304	---
3	Final heading	15,000	0	2000	4000	203 to 304	---
4	Ordinary	3,000	-3000	1500	4000	135 to 203	---
5	Ordinary	-1,500	4000	1000	3000	110 to 135	---
6	Final heading	-4,500	4000	1000	1500	110 to 135	300
7	Final heading (touchdown)	0	0	0	1500	110 to 110	350

TABLE II. — 4-D COMMAND SEQUENCE FOR EXAMPLE TRAJECTORY

[T_{min}/T_{max} to WP6 = 252/324 sec; T_{min}/T_{max} to WP7 = 295/376 sec]

Command sequence number	t, sec	States						Controls		
		x, ft	y, ft	h, ft	ψ, deg	V_g, ft/sec	V_a, ft/sec	$\dot{V}_a$, ft/sec^2	R, ft	γ, deg
1	0	1,000	0	0	0	110	135.0	1.5	Straight	6.2
2	18.2	3,249.6	0	243.2	0	137.7	162.3	1.5	4000 right	6.2
3	39.6	6,187.8	1285.9	600	47.3	174.1	191.1	1.5	Straight	3.6
4	64.1	9,383.6	4749.1	898.3	47.3	210.9	227.9	0	Straight	3.6
5	76.0	11,091.6	6594.7	1057.5	47.3	210.9	227.9	0	4000 left	3.6
6	143.7	15,000	0	2000	-166.0	252.1	227.8	0	Straight	-2.0
7	159.8	11,071.9	-979.4	1860.2	-166.0	252.1	227.8	-1.5	Straight	-2.0
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
14	300.3	-45,000	0	590.0	0	91.1	116.1	1.5	Straight	-7.5
15	300.5	-4,482.5	0	587.7	0	91.1	116.4	0	Straight	-7.5
16	345.4	-378.2	0	49.6	0	91.1	116.4	-1.5	Straight	-7.5
17	350.0	0	0	0	0	85.0	110.0	0	Straight	0

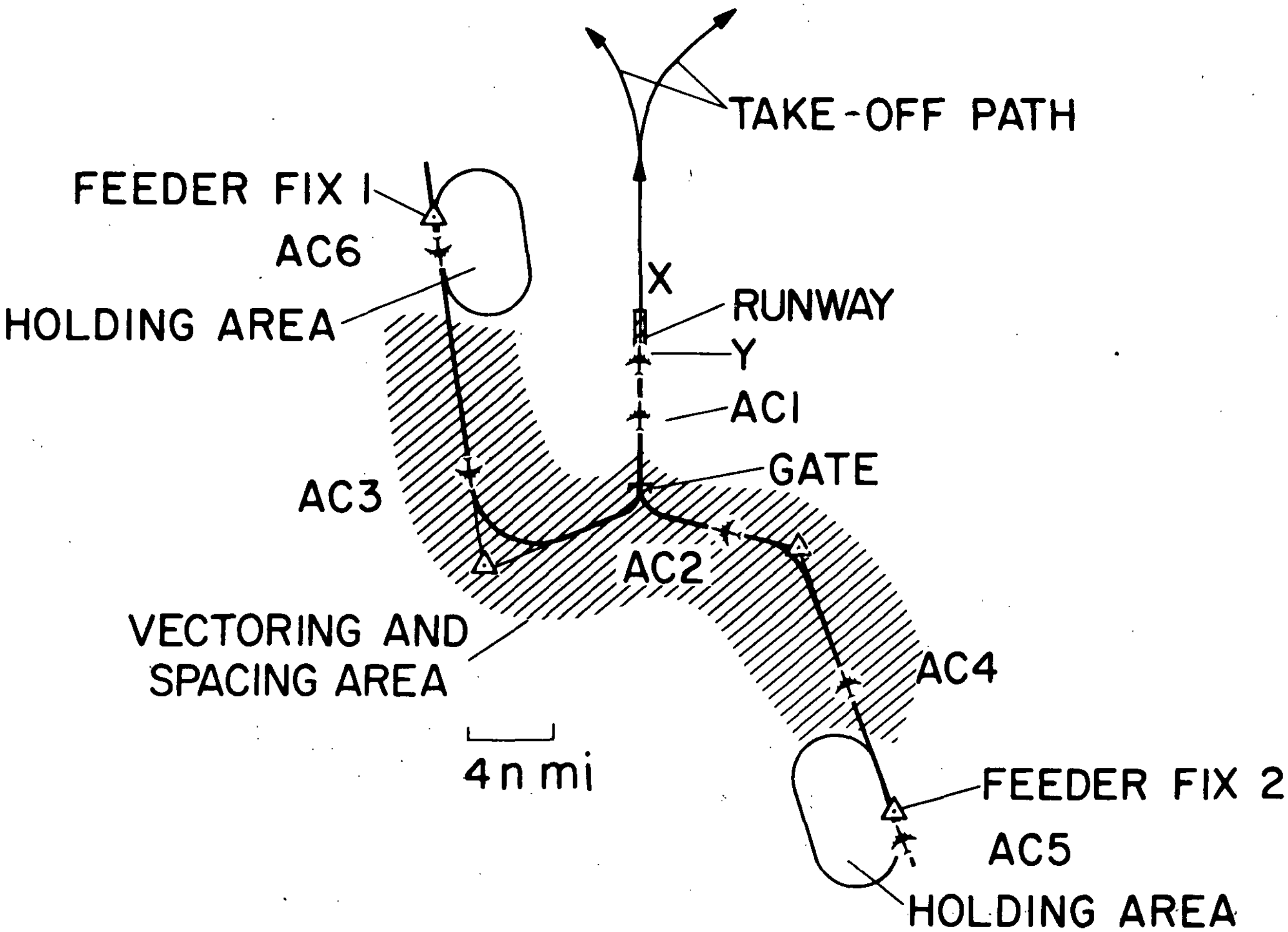

Fig. 1. Typical terminal area route structure

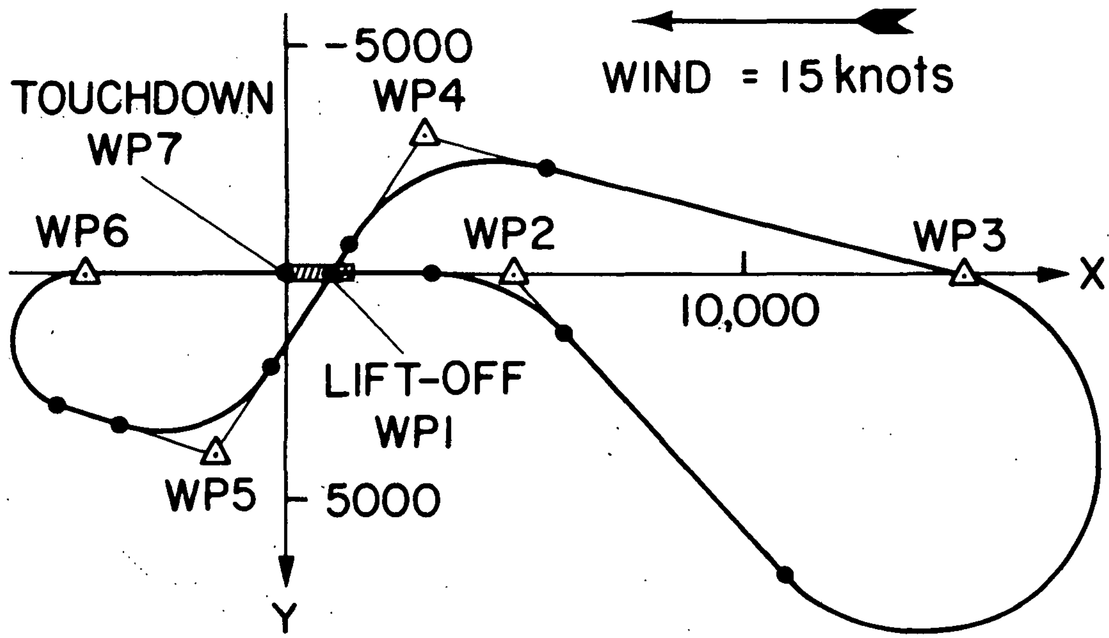

Fig. 2. Example trajectory

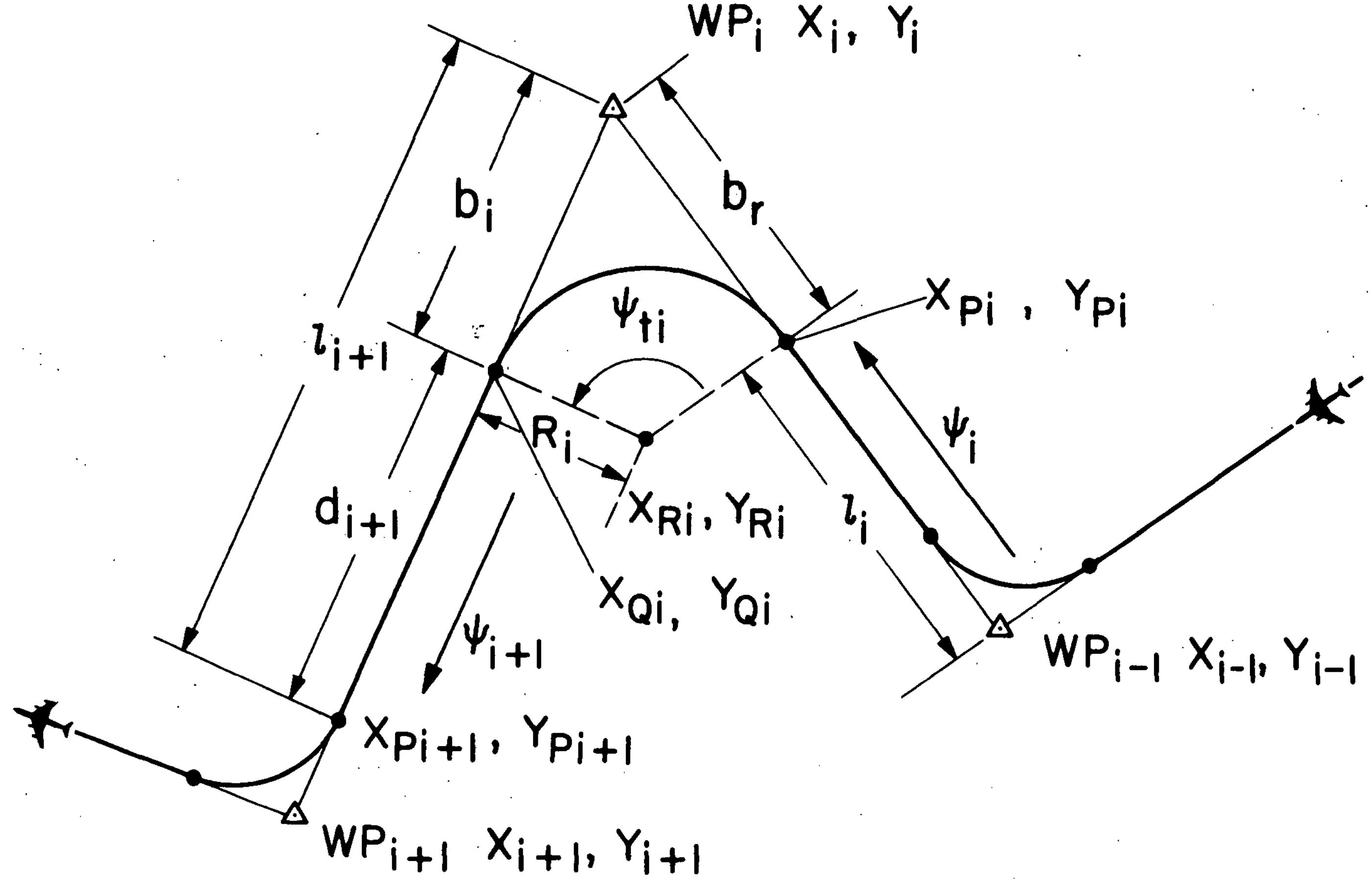

Fig. 3. Trajectory construction for ordinary waypoint

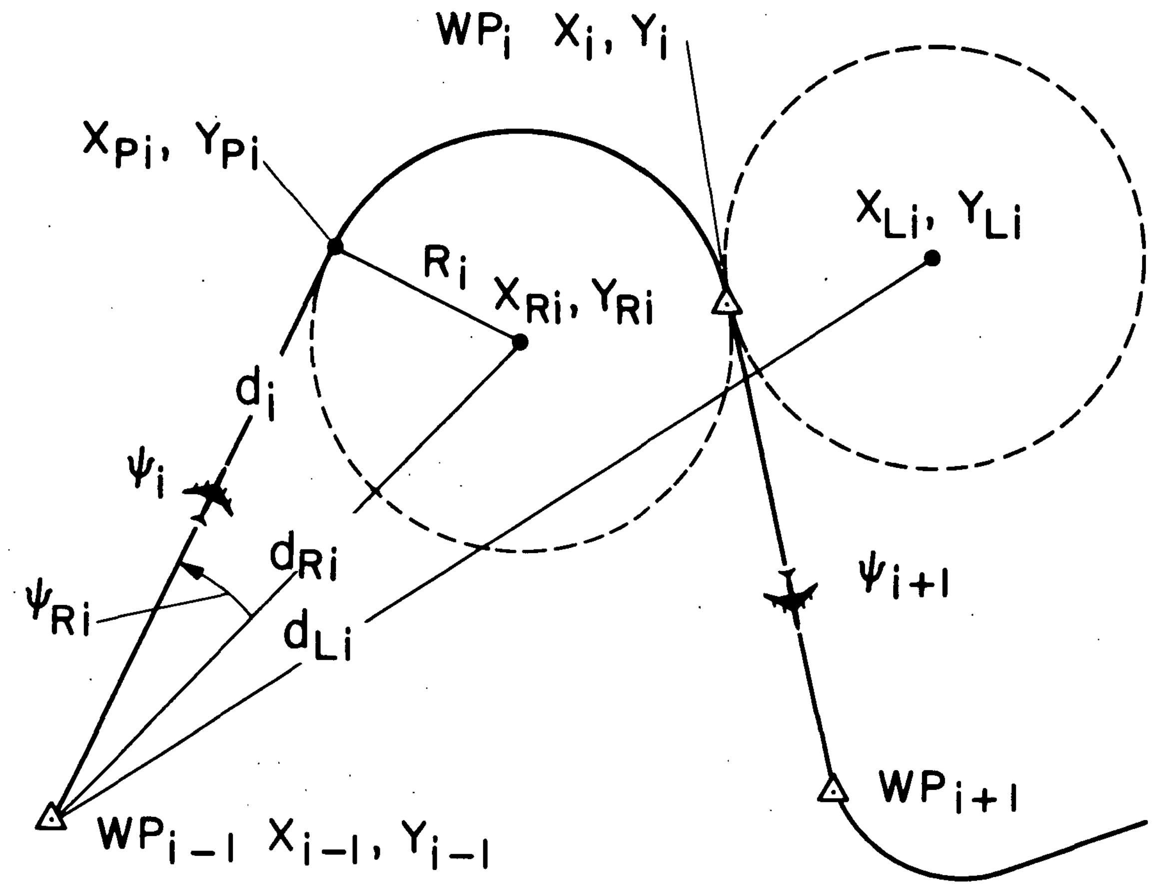

Fig. 4. Trajectory construction for final heading waypoint

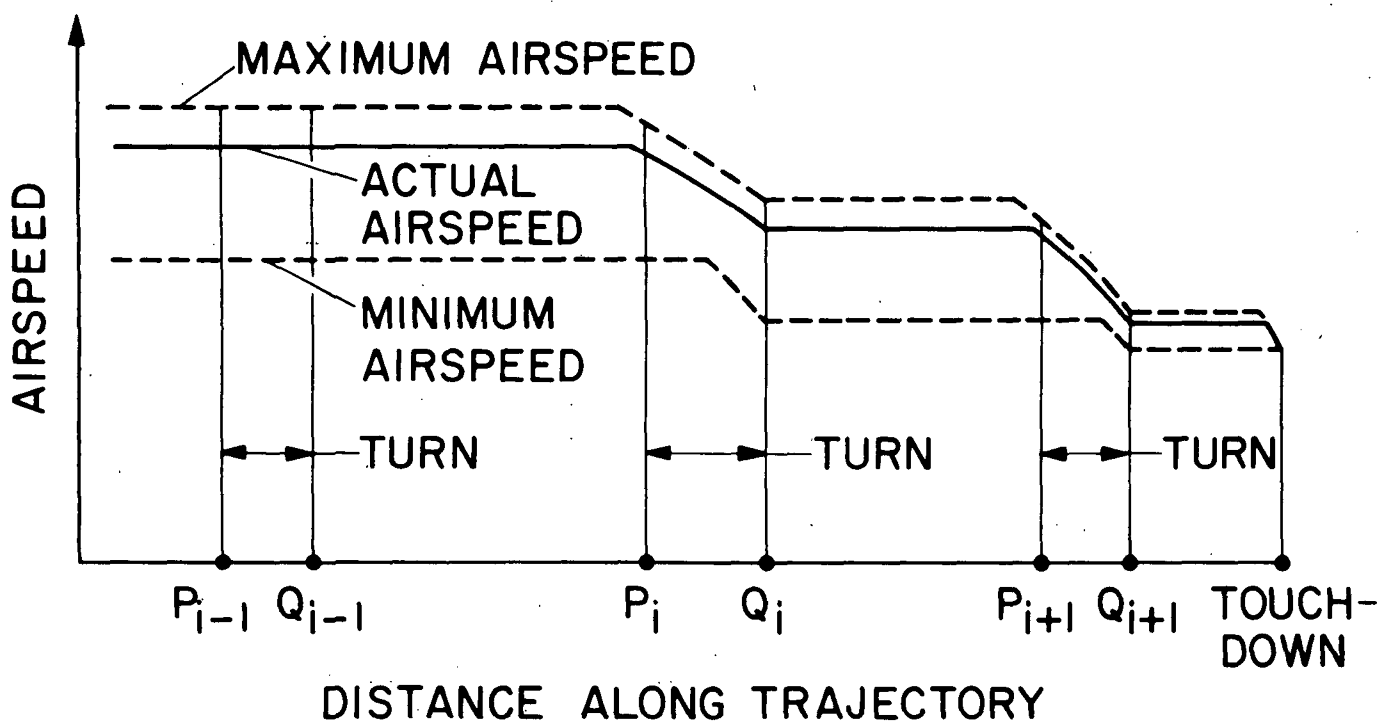

Fig. 5. Typical airspeed profile

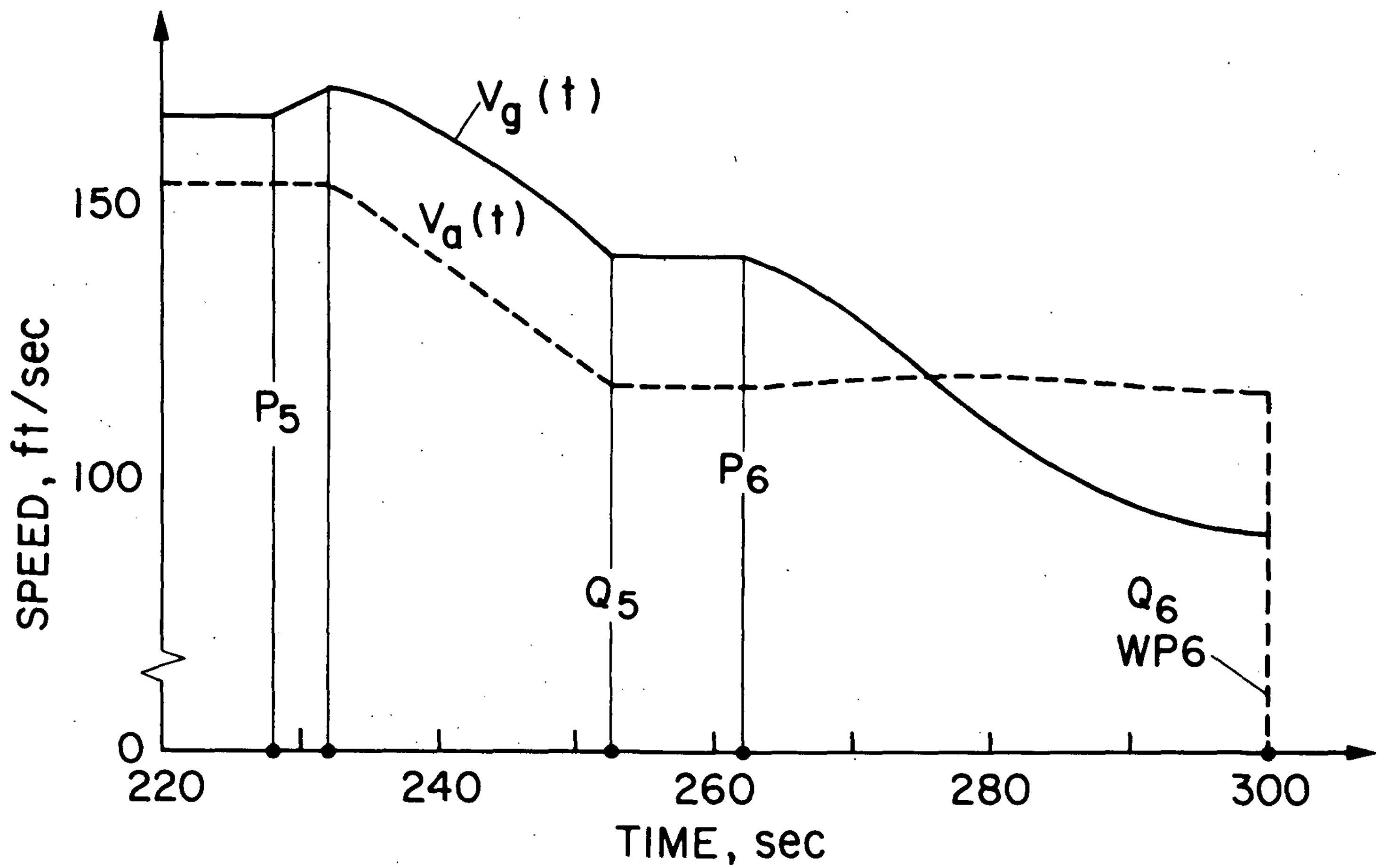

Fig. 6. Portion of airspeed and groundspeed time history for example trajectory, wind = 25 ft/sec

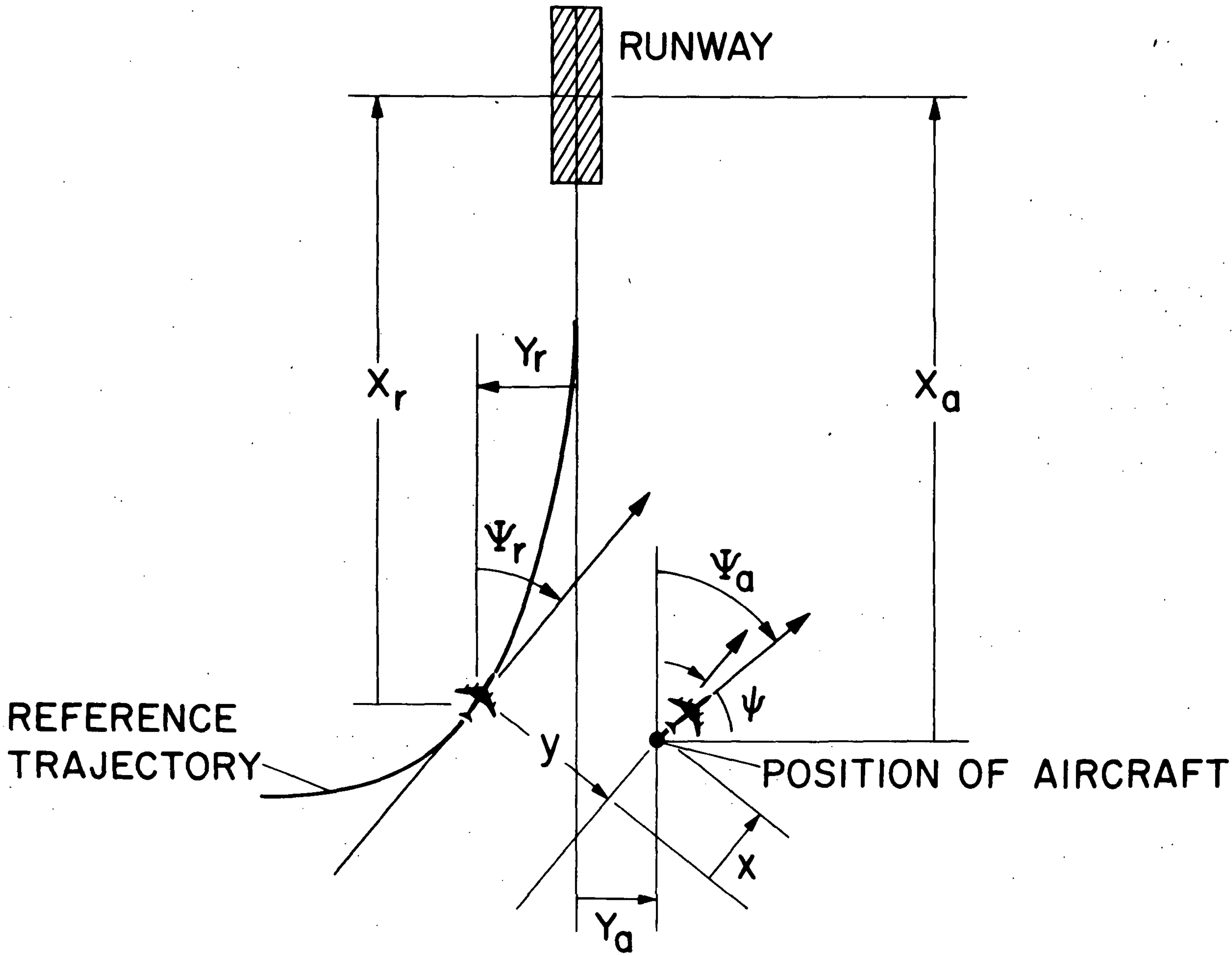

Fig. 7. Moving target reference system

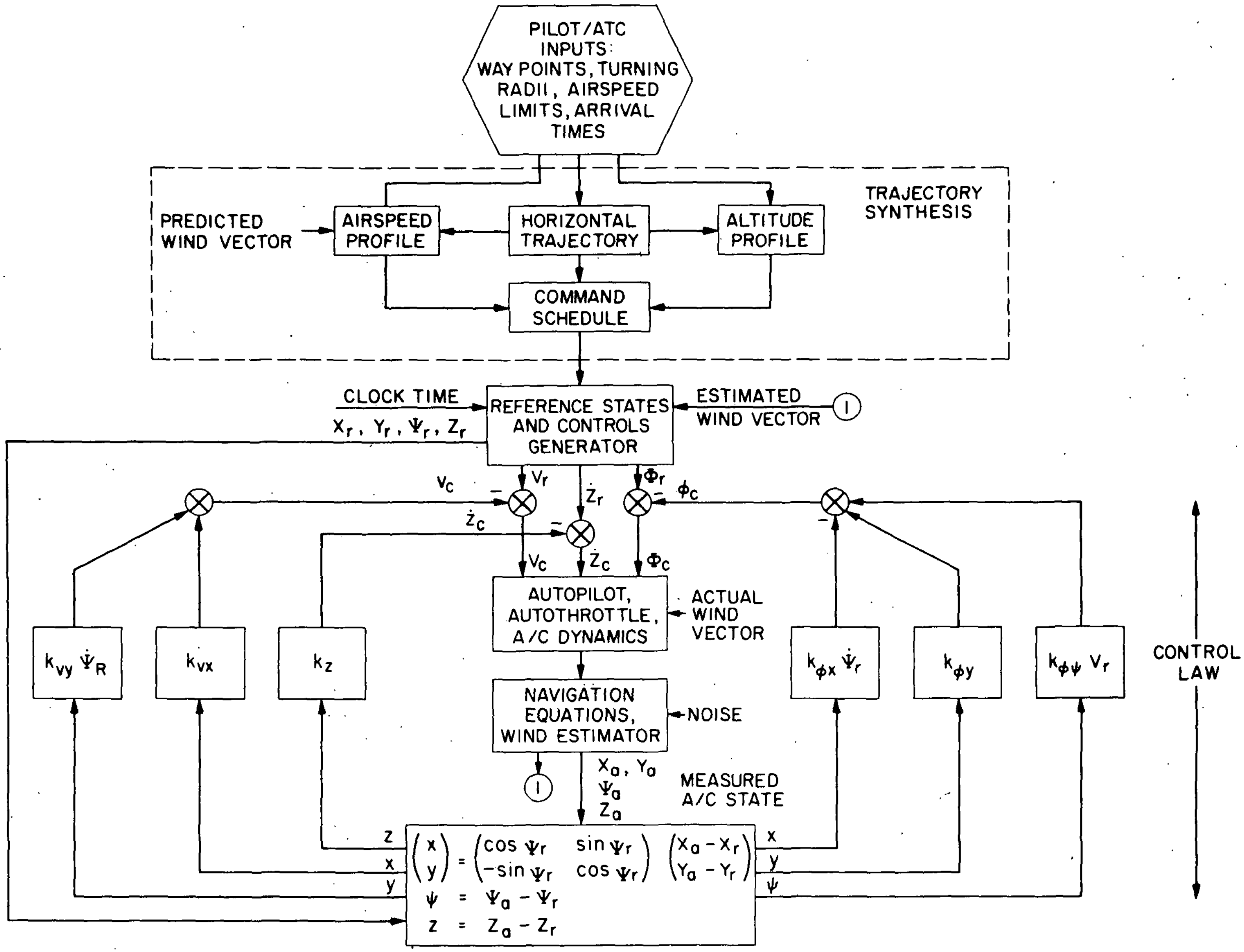

Fig. 8. Block diagram of 4-D guidance system simulation

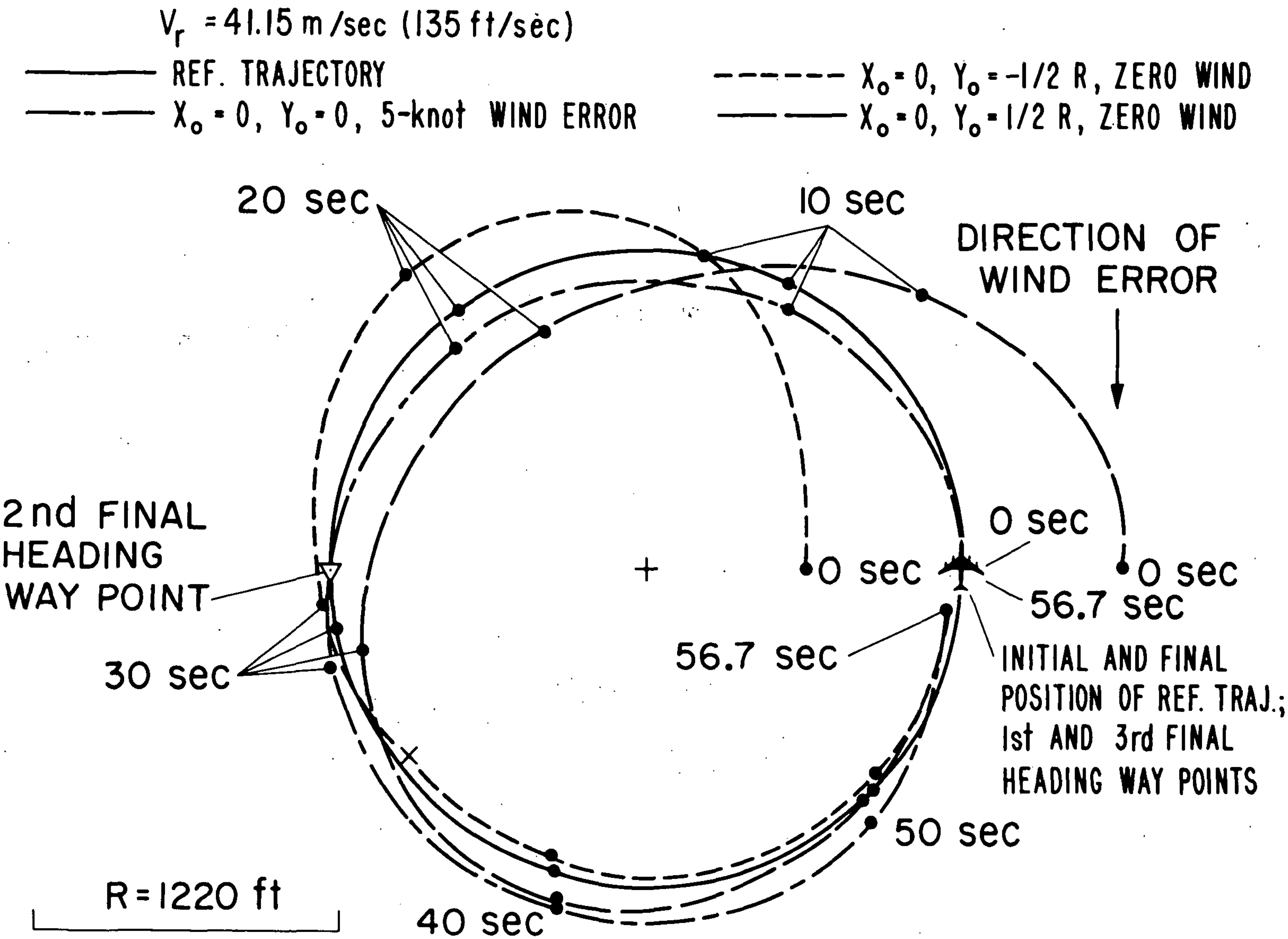

Fig. 9. Simulated flight with initial condition errors and wind estimate error